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Heat transfer and pressure drop in fractal tree-like microchannel nets

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Abstract

Inspired by the fractal pattern of mammalian circulatory and respiratory systems, a new design of fractal branching channel net for cooling of electronic chips is studied in this paper. A comparison of the new design with the traditional parallel net shows that the new fractal branching channel net has a stronger heat transfer capability and requires a lower pumping power.

1. Introduction

After the publication of Tuckerman and Pease's pioneering work [1], there has been a great deal of interest in studying fluid flow and heat transfer characteristics in micro-channels (i.e. channels with less than one millimeter in diameter) for electronic cooling applications. However, most of the existing investigations are concerned with heat transfer and fluid flow characteristics in a single micro-channel [2–4], and only a few studies [5–7] have been carried out for a network of channels as required in practical applications. With the increasing miniaturization of electronic chips and increasingly larger heat dissipation rates, better designs of cooling system are needed.

Since natural systems often give the best solution for many problems, the structure of the living organism may provide the inspiration for the design of an effective micro-cooling system for electronic chips. As discussed by Antonets et al. [8], the brain and the heart muscle would be ruined without oxygen over a time of $\tau \approx 100$ s. If the diffusion coefficient $D_c \approx 10^{-8}$ m^2/s, then the transport phenomenon occurs essentially at a distance of $\sqrt{\gamma} \approx \sqrt{\tau D_c} \approx 10^{-3}$ m which is in micro-scale. In reality the distance between cell and the capillary vessel is only 10^{-2} mm. Thus, an intercellular transport system must have a special structure that gives efficient mass transport characteristics. Since heat transfer is analogous to mass transfer, the structure of the mammalian circulatory and respiratory system can be used as a model for the design of a micro heat exchanger.

Bejan and Errera [5] first discussed about the tree network for electronic cooling application, and investigated the architecture of the volume-to-point path such that the flow resistance is minimal. They found that a tree network has the minimal resistance path. Bejan [6] showed that the total heat convected by a double tree is proportional to the total volume raised to a power of 3/4. Kearney [9] pointed out that equipment built with fractal characteristics could offer advantages over traditional fluid mixers and distributors. Pence [7] investigated the cooling of a circular heated surface by fractal-like branching channel networks, and compared it with parallel channels. In her work, a fractal dimension of 2 was studied.

In this paper, we will study both the heat transfer and pressure drop characteristics in fractal-tree nets for the cooling of a micro-electronic chip, which is usually of rectangular shape. A comparison of the cooling capacity and the power requirement of the fractal tree channel net with the traditional parallel channels is made.

2. A fractal branch net of rectangular shape

Fig. 1 shows a branching vessel tree system of the arteries or veins in human beings and animals in which

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Nomenclature

c	constant
D	fractal dimension of channel length distribution
d	diameter of parallel channels
D_c	diffusion coefficient
d_k	branch diameter at the kth level
f	friction factor
h_k	heat transfer coefficient of the kth level channel
L	length of parallel channel
L_k	branch length of the kth level
m	total number of branching levels
N	number of branches into which a single channel bifurcates
n	number of parallel channels
P	pumping power of fractal channels
P_{p2}	pumping power of parallel channels
Q	flow rate
Q_h	total convective heat transfer of fractal channels
Q_{hp1}	total convective heat transfer of parallel channels with the same heat transfer area as fractal channels and diameter of d_0
Q_{hp2}	convective heat transfer of parallel channels, equal to Q_h
Q_{p2}	flow rate of parallel channels
r	diffusion area
S	total heat transfer area of the fractal channel net
S_{p2}	total heat transfer area of parallel channels
v_0	velocity in the initial channel
β	ratio of the diameter of the channel at the $(k+1)$th branch level versus the diameter of the channel at the kth branch level
γ	ratio of the length of the channel at the $(k+1)$th branch level versus the length of the channel at the kth branch level
τ	time
Δ	fractal dimension of the hydraulic diameter distribution
Δp	pressure drop
ΔT	temperature difference

the blood circulates [8]. Such a dendroid structure consists of subsequent branchings (bifurcations) and foldings. The human arterial vessels are known to have about 30 branches from the aorta to the arterioles [8]. The higher- and lower-level branching structures are similar and the structure lacks a characteristic length.

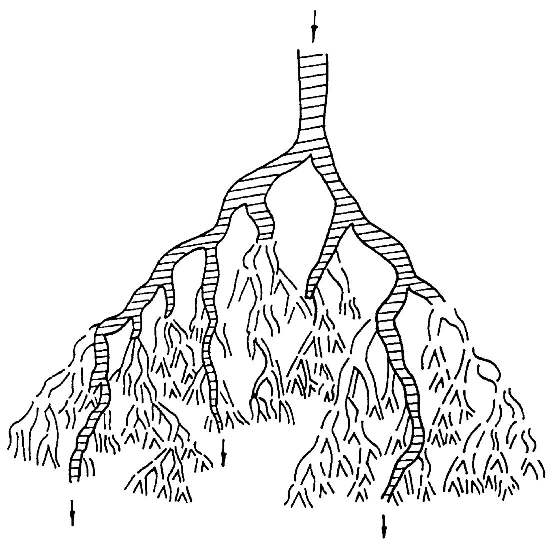

Fig. 1. Branching vessel tree of arteries or veins [8].

This property is known as self-similarity, and this structure is known as a fractal [10]. The fractal network branching results in an increased number of channels with smaller diameter, and an increase in the total cross-sectional flow area. As mentioned earlier, the mass transfer is efficient in these fractal network systems.

We now discuss the generation of a fractal branch net for the cooling of electronic chips of rectangular shape. Suppose that every channel is divided into two branches at the next level as shown in Fig. 2, i.e., a single channel bifurcates and $N = 2$. Since a micro-electronic chip is often of rectangular shape, the branching angle ϕ is π (see also Fig. 2). We define that the ratio of length of the channel at the $(k+1)$th branching level to the length of the channel at the kth branching level as

$$\gamma = \frac{L_{k+1}}{L_k} \tag{1}$$

and therefore,

$$L_k = L_0\gamma^k, \tag{2}$$

where L_0 is the length at the 0th branching level. Note that the fractal dimension D is generated from [10]:

$$N = \gamma^{-D} \tag{3}$$

so that,

$$\gamma = N^{-1/D}. \tag{4}$$

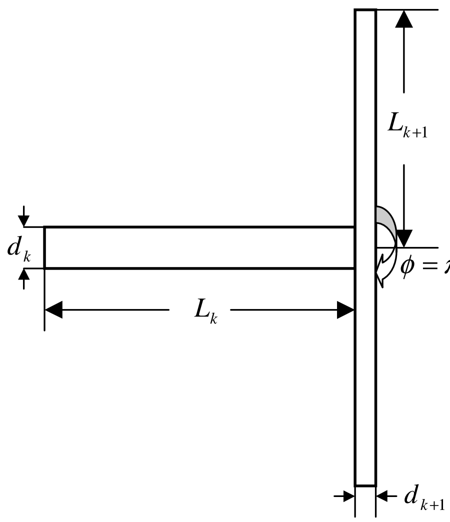

Fig. 2. Stick and two branches.

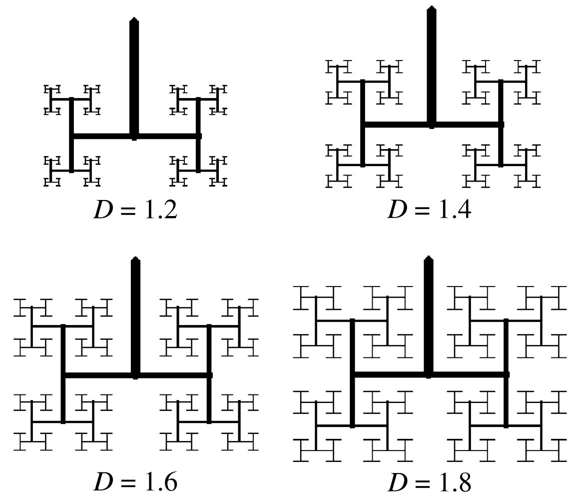

Fig. 3. Branch nets having seven branching levels at different fractal dimensions.

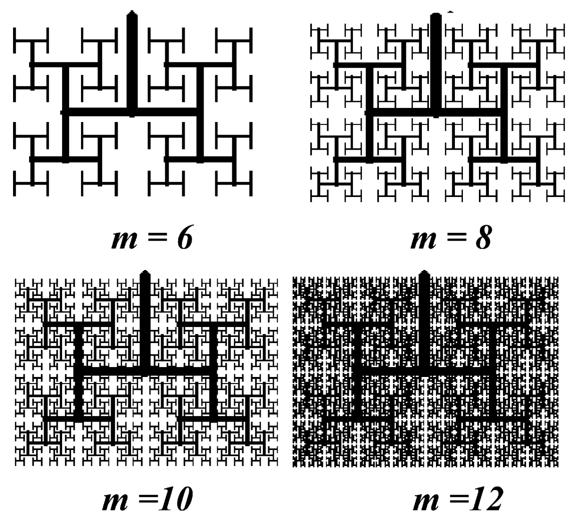

Fig. 4. Branch profiles of different total branching levels at $D = 2$.

If a botanical tree's branch hydraulic diameters before and after bifurcation are given by d_k and d_{k+1}, it satisfies the relation [10]

$$d_k^{\Delta} = N d_{k+1}^{\Delta}, \tag{5}$$

which gives

$$N = \left(\frac{d_{k+1}}{d_k}\right)^{-\Delta}, \tag{6}$$

where Δ is the fractal dimension of the hydraulic diameter distribution. The hydraulic diameter ratio is defined as

$$\beta = \frac{d_{k+1}}{d_k} = N^{-1/\Delta}. \tag{7}$$

Thus,

$$d_k = d_0 \beta^k. \tag{8}$$

On the basis of above relation and starting with an initiator channel, the skeleton of the model grows by recursive application of a growth-and-branching generator to the model's tips. Fig. 3 shows the branch nets having a total of seven branching levels ($m = 7$ with m denoting the total number of branching levels, exclusive of the 0th level) at different fractal dimensions ($D = 1.2$, 1.4, 1.6, 1.8) with the same initiator. It is shown that the size of the rectangular area occupied by the channels having the same seven branching levels is slightly different for different values of D, and the density of the channel increases with increasing value of fractal dimension D. Fig. 4 gives the channel density of different total branching levels ($m = 6, 8, 10, 12$) at $D = 2$. From this figure, we can see that the rectangular area is full of channels with increasing value of m at $D = 2$.

In order to have free circulation of the cooling fluid and a uniform heat transfer, the micro heat exchanger is designed to have a top and a bottom circulation pattern in a wafer. The bottom has the same distribution of the channel net as the top one except that the inlet and the outlet channels are pointed toward opposite direction (see Fig. 5(a)). The channel pattern is shown in Fig. 5(b). The channel of highest branching level ($k = m$) on the top communicates with the channel of highest branching level on the bottom, as shown in Fig. 5(c). This design is to facilitate measurements of pressure drop and heat transfer to be carried out for the micro-channel net-like fractal trees for the verification of the present theory.

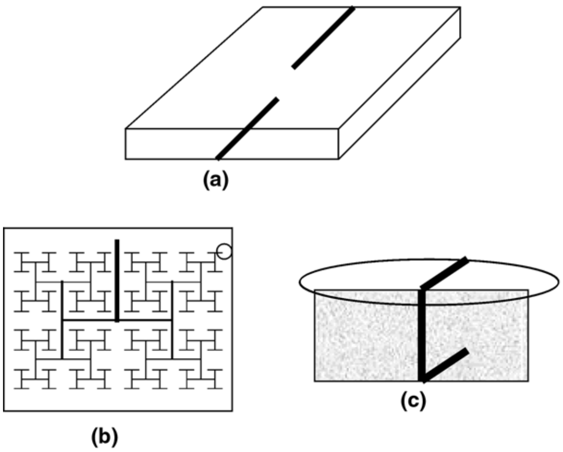

Fig. 5. Sketch of a micro heat exchanger.

3. Heat transfer in the fractal branch net

The total heat transfer area of the channel network S with m branching levels is given by

$$S = 2\sum_{k=0}^{m} S_k = 2\sum_{k=0}^{m} \pi d_k L_k N^k, \tag{9}$$

where the factor 2 indicates that both the upside and the downside are considered (see Fig. 5(a)). Substituting Eqs. (2) and (8) into Eq. (9) gives

$$S = 2\sum_{k=0}^{m} \pi d_0 \beta^k L_0 \gamma^k N^k = 2\pi d_0 L_0 \frac{1-(N\beta\gamma)^{m+1}}{1-N\beta\gamma}. \tag{10}$$

Assuming that the flow through each channel is laminar and fully developed both thermally and hydrodynamically, the Nusselt number remains constant through each level of the branching. It follows that the heat transfer coefficient of the higher-level branching will increase as

$$\frac{h_{k+1}}{h_k} = \frac{d_k}{d_{k+1}}. \tag{11}$$

Substituting Eq. (7) into Eq. (11) yields

$$\frac{h_{k+1}}{h_k} = \frac{1}{\beta}. \tag{12}$$

Consequently,

$$h_k = h_0 \beta^{-k}. \tag{13}$$

A fully developed flow with constant heat flux in a uniform cross-section yields a constant temperature difference ΔT between the wall surface and bulk flow. In order to compare with the parallel channels, the temperature difference for each branching level is also assumed to be the same constant ΔT. Thus, the total convective heat transfer will be

$$Q_\mathrm{h} = 2\sum_{k=0}^{m} h_k S_k \Delta T = 2\sum_{k=0}^{m} h_k \pi d_k L_k N^k \Delta T. \tag{14}$$

Substituting Eqs. (2), (8) and (13) into Eq. (14), we can get

$$Q_\mathrm{h} = 2\sum_{k=0}^{m} h_0 \beta^{-k} \pi d_0 \beta^k L_0 \gamma^k N^k \Delta T = 2\pi d_0 L_0 h_0 \frac{1-(N\gamma)^{m+1}}{1-N\gamma} \Delta T. \tag{15}$$

For a parallel channel net with diameter d_0, with the same heat transfer area, the same temperature difference and Nusselt number, the total convective heat transfer is

$$Q_\mathrm{hp1} = h_0 S \Delta T. \tag{16}$$

Substituting Eq. (10) into Eq. (16) gives

$$Q_\mathrm{hp1} = 2\pi d_0 L_0 h_0 \frac{1-(N\beta\gamma)^{m+1}}{1-N\beta\gamma} \Delta T. \tag{17}$$

Thus, the ratio of $Q_\mathrm{h}/Q_\mathrm{hp1}$ is

$$Q_\mathrm{h}/Q_\mathrm{hp1} = \frac{\left[1-(N\gamma)^{m+1}\right](1-N\beta\gamma)}{\left[1-(N\beta\gamma)^{m+1}\right](1-N\gamma)}, \tag{18}$$

where γ is given by Eq. (4) which is a function of D and N, while β is given by Eq. (7) which is a function of Δ and N. Eq. (18) was computed for $N = 2$, $\Delta = 3$, $m = 3, 4, 5$, and $D = 1.1$–2, and the results are presented in Fig. 6. Note that the value of $\Delta = 3$ was chosen for computation because the fractal dimension Δ of a lung bronchial tree is 3 according to Mandelbrot [10]. It is

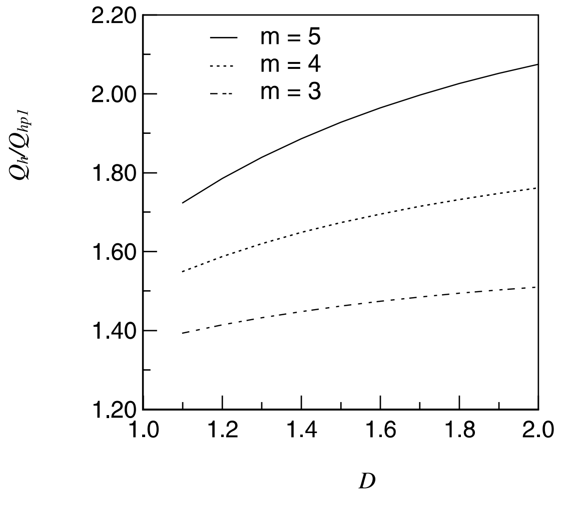

Fig. 6. Plot of $Q_\mathrm{h}/Q_\mathrm{hp1}$ versus D at $\Delta = 3$, $n = 2$ and $m = 3, 4, 5$.

seen from Fig. 6 that: (1) the branch net has a better heat transfer capability than those of the traditional parallel channels, and (2) the larger of the fractal dimension D or the total number of branching levels m, the larger heat transfer capability.

4. Pressure drop in the fractal branch net

Neglecting the effect of bifurcation, the pressure drop of a laminar fully developed flow in the fractal branch net is

$$\Delta p = 2\sum_{k=0}^{m} \frac{c}{d_k^2} L_k v_k, \tag{19}$$

where v_k is the velocity in the kth branching level channel and c is a constant. From the conservation of mass, we have

$$v_k d_k^2 N^k = v_0 d_0^2 \tag{20}$$

and therefore

$$v_k = v_0 \left(\frac{d_0}{d_k}\right)^2 \frac{1}{N^k}. \tag{21}$$

Substituting Eq. (21) into Eq. (19) gives

$$\Delta p = 2\sum_{k=0}^{m} \frac{c}{d_k^4} L_k d_0^2 v_0 \frac{1}{N^k}. \tag{22}$$

The flow rate Q is

$$Q = v_0 \frac{\pi}{4} d_0^2 \tag{23}$$

while the required pumping power is

$$P = Q\Delta p. \tag{24}$$

Substituting Eqs. (22) and (23) into Eq. (24) gives

$$P = 2v_0 \frac{\pi}{4} d_0^2 \sum_{k=0}^{m} \frac{c}{d_k^4} d_0^2 L_k v_0 \frac{1}{N^k}. \tag{25}$$

Substituting Eqs. (2) and (8) into Eq. (25) yields

$$P = \frac{\pi}{2} v_0 d_0^2 \sum_{k=0}^{m} \frac{c}{(d_0\beta^k)^4 N^k} d_0^2 L_0 \gamma^k v_0$$
$$= \frac{\pi}{2} v_0^2 c L_0 \frac{1 - \left[(\gamma/(N\beta^4))\right]^{m+1}}{1 - \gamma/(N\beta^4)}. \tag{26}$$

Next, let us compare the pressure drop in a fractal net with the traditional parallel net. The parallel channel net has the same velocity as v_0, the same temperature difference and heat transfer rate with the fractal channel net. Thus,

$$S_{p2} = L\pi d n, \tag{27}$$

$$Q_{hp2} = hS_{p2}\Delta T = Q_h, \tag{28}$$

where S_{p2} is the heat transfer area of the parallel network, d is the diameter, $L = 2L_0$, and n is the number of channels which is assumed to be an integer. Substituting Eqs. (15) and (27) into Eq. (28) gives

$$hL\pi d n = 2\pi d_0 L_0 h_0 \frac{1 - (N\gamma)^{m+1}}{1 - N\gamma}. \tag{29}$$

Assuming that the flow in parallel channel has the same Nusselt number as those in the fractal tree channel, so $hd = h_0 d_0$. It follows from Eq. (29) that

$$n = \frac{1 - (N\gamma)^{m+1}}{(1 - N\gamma)}. \tag{30}$$

The pressure drop is

$$\Delta p_{p2} = 2\frac{c}{d^2} L_0 v_0. \tag{31}$$

The flow rate is

$$Q_{p2} = \frac{\pi}{4} v_0 d^2 n \tag{32}$$

while the required pumping power is

$$P_{p2} = Q_{p2}\Delta p_{p2}. \tag{33}$$

Substituting Eqs. (31) and (32) into Eq. (33) gives

$$P_{p2} = \frac{\pi}{2} c L_0 v_0^2 n. \tag{34}$$

Eq. (34) with the aid of Eq. (30) yields

$$P_{p2} = \frac{\pi}{2} c v_0^2 L_0 \frac{1 - (N\gamma)^{m+1}}{(1 - N\gamma)}. \tag{35}$$

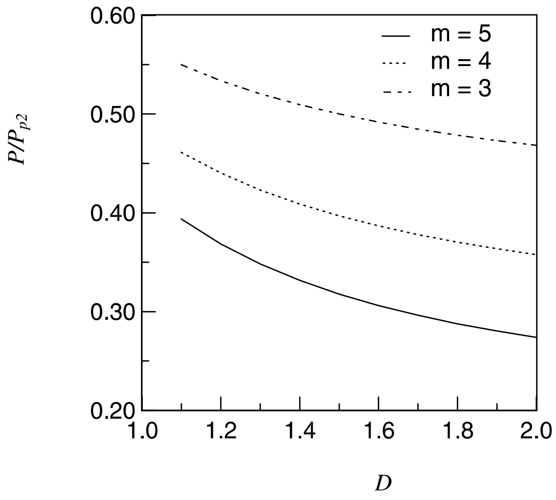

Fig. 7. Plot of P/P_{p2} versus D at $\Delta = 3$, $n = 2$ and $m = 3, 4, 5$.

It follows that the ratio P/P_{p2} is

$$P/P_{p2} = \frac{\left[1 - \left(\gamma/(N\beta^4)\right)^{m+1}\right](1 - N\gamma)}{\left[1 - \gamma/(N\beta^4)\right]\left[1 - (N\gamma)^{m+1}\right]}. \quad (36)$$

Eq. (36) was computed for $N = 2$, $\Delta = 3$, $m = 3, 4, 5$, and $D = 1.1$–2. The results are presented in Fig. 7, which shows that the required pumping power of the fractal net is much lower than that of the parallel net. As the fractal dimension D or the total number of branching levels m becomes larger, a lower pumping power is required.

5. Concluding remarks

In this paper, fractal branching channel nets with different dimensions D and different total branching levels m for the design of a micro heat exchanger of rectangular shape are investigated. A comparison of heat transfer and pressure drop is made between fractal branching channel nets with the traditional parallel channel net. The comparison is based on the following two assumptions: (1) the flow is laminar and fully developed, and (2) the effect of bifurication on pressure drop is negligible. It is found that the fractal net can increase the total heat transfer rate while it reduces the total pressure drop in the fluid. A larger fractal dimension D or a larger total number of branching levels m is found to have a stronger heat transfer capability with a smaller pumping power required. Thus, the fractal branching channel net enhances the efficiency of a micro heat exchanger. We envisage that this net-like fractal tree design of micro-channels will find a wide range of applications in Micro-Electro-Mechanical-Systems (MEMS), bioengineering and biotechnology, aerospace, and selective membranes, etc. in the future.

Acknowledgements

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